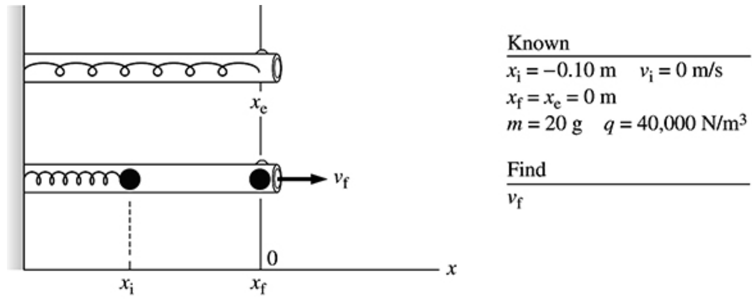
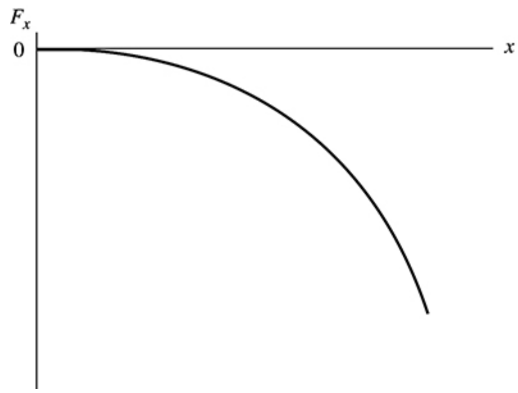
11.59. Model: A "sprong" that obeys the force law $F_x = -q(x - x_e)^3$, where q is the sprong constant and x_e is the equilibrium position.

Visualize: We place the origin of the coordinate system on the free end of the sprong, that is, $x_e = x_f = 0$ m.



Solve: **(a)** The units of q are N/m^3.

(b) A cubic curve rises more steeply than a parabola. The force increases by a factor of 8 every time x increases by a factor of 2.



(c) Since $F_x = -\dfrac{dU}{dx}$, we have $U(x) = -\int F_x dx = -\int_0^x (-qx^3)dx = \dfrac{qx^4}{4}$.

(d) Applying the energy conservation equation to the ball and sprong system:

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + 0\text{ J} = 0\text{ J} + \frac{qx_i^4}{4}$$

$$\Rightarrow v_f = \sqrt{\frac{q}{m}\frac{x^4}{2}} = \sqrt{\frac{(40{,}000\text{ N/m}^3)}{(0.020\text{ kg})}\cdot\frac{(-0.10\text{ m})^4}{2}} = 10\text{ m/s}$$